APRIL/MAY 2023

DMA33 — DIFFERENTIAL GEOMETRY

Time : Three hours   Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

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1. Write down the Serret – Frenet formulae.

2. Define Osculating plane.

3. Write down the intrinsic properties.

4. What are the types of helicoids?

5. Define Gaussian curvature.

6. State Minding's theorem.

7. Give the necessary and sufficient condition for a surface to be developable is that its Gaussian curvature shall be zero.

8. What do you mean by characteristic line?

9. Write down the Gauss equation.

10. State the fundamental existence theorem for surfaces.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Find the curvature and torsion of the cubic curve $r = \left(u, u^2, u^3\right)$

Or

(b) Show that $\left[r', r'', r'''\right] = k^2 \tau$.

12. (a) Obtain surface of revolution which is isometric with a region of the right helicoid.

Or

(b) Find the coefficients of the direction which makes an angle $\frac{\pi}{2}$ with the direction whose coefficients are (l, m).

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13. (a) Compute the geodesic on a surface of revolution.

Or

(b) Show that the curve of family $\frac{v^3}{u^2} = constants$ are geodesics on a surface with metric $v^2 du^2 - 2uv\, du\, dv + 2u^2 dv^2 \left(u > 0, v > 0\right)$.

14. (a) Obtain the Gaussian curvature.

Or

(b) Derive the Rodrigues formula.

15. (a) Prove that the only compact surface of class ≥ 2 for which every point is an umbilic are spheres.

Or

(b) Show that the only compact surfaces whose Gaussian curvature is positive and mean curvature constant are spheres.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Derive the curvature and torsion of a curve given as the intersection of two surfaces.

17. On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z = \text{constant}$.

18. Show that every helix on a cylinder is geodesic.

19. Obtain the second fundamental form.

20. State and prove Hilbert lemma.
